Mixing Probabilistic Meteorology Outlooks in Operational Hydrology

By Thomas E. Croley II[1]

Abstract: There are now several kinds of probabilistic meteorology outlooks available to the water resource engineer or hydrologist. These outlooks are defined over different time periods at different lag times, and they forecast either event probabilities or only most-probable events. An existing operational hydrology approach (for making hydrology outlooks) builds a set of hydrological possibilities from past meteorology to match forecast event probabilities, but it does not consider most-probable event forecasts. This approach is extended to mix both types of probabilistic meteorology outlooks in determining weights to apply to the set of hydrological possibilities to make hydrological outlooks. Boundary condition equations for the weights are constructed corresponding to forecast event probabilities, and boundary condition inequalities are constructed corresponding to forecast most-probable events. The inequalities are converted to equivalent equations through the introduction of additional variables. The resulting set of all boundary condition equations is solved for physically relevant values. The solution is an optimization problem for the general case, similar to earlier consideration of only forecast event probabilities. An example illustrates the concepts and methods.

PROBABILISTIC METEOROLOGY OUTLOOKS

There are now several kinds of probabilistic meteorology outlooks available to the water resource engineer or hydrologist. The National Oceanic and Atmospheric Administration (NOAA) Climate Prediction Center provides a monthly climate outlook at midmonth, consisting of a one-month outlook for the next (full) month and 13 three-month outlooks, going into the future in overlapping fashion in one-month steps. Each outlook estimates probabilities of average air temperature and total precipitation falling within the lower, middle, and upper thirds of observations from 1961–90. The Climate Prediction Center also produces a 6–10 day outlook, covering the five-day period beginning six days hence. It predicts which of five intervals of five-day average air temperature are expected; less than the 10% quantile, between the 10% and 30% quantiles, between the 30% and 70% quantiles, between the 70% and 90% quantiles, or greater than the 90% quantile. The quantiles are defined from observations from 1961–90 (Hoopingarner, personal communication, 1996). It also predicts which of the three intervals of total precipitation are expected (lower, middle, or upper thirds of observations from 1961–90) or specifies that no precipitation is expected. The Climate and Water Information Branch of Environment Canada (EC) produces both a one-month outlook at beginning- and midmonth and a three-month outlook each quarter of average air temperature. Each outlook predicts which of three intervals (lower, middle, or upper thirds of observations from 1961–90) of one-month and three-month average air temperature are expected. Environment Canada is also considering several new outlooks, experimentally at the present time, for both temperature and precipitation over three-month periods going one year into the future in three-month steps. All of these outlooks differ in several important respects. They are defined over different time periods (five days, one month, three months) at different lag times (zero months, six days, 1/2 month, 1 1/2, 2 1/2, . . . , 12 1/2 months from when they are issued; real lags depend on when they are actually used), and they specify either a probability of falling within an interval (event probability) or only the most-probable interval (most-probable event).

[1]Res. Hydro., Great Lakes Envir. Res. Lab., 2205 Commonwealth Blvd., Ann Arbor, MI 48105-1593.

Note. Discussion open until March 1, 1998. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on October 3, 1996. This paper is part of the ***Journal of Hydrologic Engineering***, Vol. 2, No. 4, October, 1997. ©ASCE, ISSN 1084-0699/97/0004-0161–0168/$4.00 + $.50 per page. Paper No. 14229.

HYDROLOGICAL OUTLOOKS

Users of probabilistic meteorology outlooks can interpret them through an operational hydrology approach (Croley 1996, 1997) that considers historical meteorology as possibilities for the future. The approach segments the historical record and uses each segment with models to simulate a hydrological possibility for the future; see Fig. 1. Each segment of the historical record then has associated time series of meteorological and hydrological variables, representing a possible "scenario" for the future. The approach then considers the resulting set of possible future scenarios as a statistical sample and infers probabilities and other parameters associated with both meteorology and hydrology through statistical estimation from this sample; see Fig. 1. However, the relative frequencies of selected events are fixed at historical values that are incompatible (generally) with those specified in probabilistic meteorology outlooks. Only by restructuring the set of possible future scenarios can we obtain relative frequencies of selected events that match probabilistic meteorology outlooks. There are many methods for restructuring the set of possible future scenarios (Croley 1996, 1997; Day 1985; Smith et al. 1992).

Croley (1996, 1997) discusses restructuring to match forecast event probabilities as given in NOAA's monthly climate outlooks. However, his method does not address matching

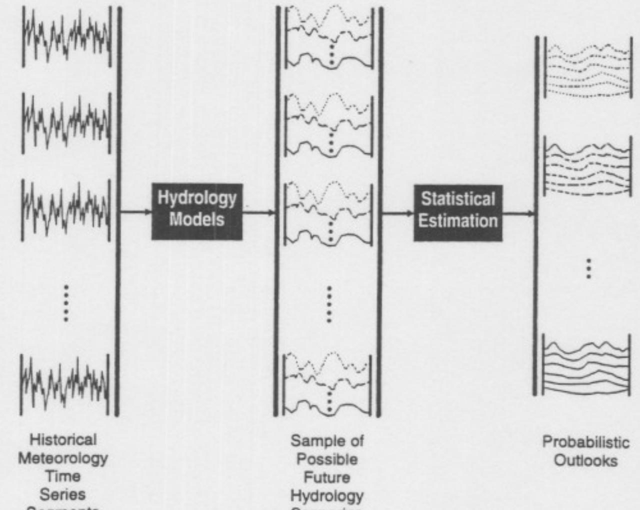

FIG. 1. Operational Hydrology Approach

most-probable event forecasts such as the NOAA 6–10 day outlook or the EC one-month and three-month outlooks. His approach is extended here to mix all of these probabilistic meteorology outlooks to make hydrological outlooks. The following two sections describe matching event probabilities and most-probable events, respectively. Methodology is then presented to mix these outlooks followed by an example and discussion.

MATCHING EVENT PROBABILITIES

Croley (1996), Day (1985), and Smith et al. (1992) provide weighted statistics defined over the set of possible future scenarios where the weights are determined to match forecast event probabilities. [See Croley (1996) for more information on the underlying concepts.] For example

$$\hat{P}[X \le x] = \frac{1}{n}\sum_{i\in\Omega} w_i; \quad \Omega \equiv \{i|x_i \le x\} \tag{1}$$

where $\hat{P}[\ ]$ denotes relative frequency (used as probability estimate); X = any variable (either historical meteorological or simulated hydrological); x = value of X; n = number of possible future scenarios (number of historical record segments); and w_i = weight to apply to ith value of X (x_i) in set of possible future scenarios. Read the set notation in (1) as "Ω is the set of all values of i such that $x_i \le x$." Note that the n weights sum to n. If $w_i = 1$, $i = 1, \ldots, n$, then (1) gives contemporary (unstructured) estimates.

As just mentioned, the weights were determined by matching relative frequencies, in (1), to the event probabilities of the NOAA Climate Outlook

$$\frac{1}{n}\sum_{i\in A_g} w_i = \hat{P}[T_g \le \tau_{g,0.333}]; \quad A_g \equiv \{i|t_{g,i} \le \tau_{g,0.333}\}; \quad g = 1, \ldots, 14 \tag{2a}$$

$$\frac{1}{n}\sum_{i\in B_g} w_i = \hat{P}[T_g > \tau_{g,0.667}]; \quad B_g \equiv \{i|t_{g,i} > \tau_{g,0.667}\}; \quad g = 1, \ldots, 14 \tag{2b}$$

$$\frac{1}{n}\sum_{i\in C_g} w_i = \hat{P}[Q_g \le \theta_{g,0.333}]; \quad C_g \equiv \{i|q_{g,i} \le \theta_{g,0.333}\}; \quad g = 1, \ldots, 14 \tag{2c}$$

$$\frac{1}{n}\sum_{i\in D_g} w_i = \hat{P}[Q_g > \theta_{g,0.667}]; \quad D_g \equiv \{i|q_{g,i} > \theta_{g,0.667}\}; \quad g = 1, \ldots, 14 \tag{2d}$$

$$\frac{1}{n}\sum_{i=1}^{n} w_i = 1 \tag{2e}$$

where T_g and Q_g = average air temperature and total precipitation, respectively, over period g (g = 1 corresponds to one-month period, and $g = 2, \ldots, 14$ corresponds to 13 successive three-month periods, each overlapping by one month); $\tau_{g,\gamma}$ and $\theta_{g,\gamma}$ =, respectively, temperature and precipitation reference γ-probability quantiles for period g; and $t_{g,i}$ and $q_{g,i}$ = average air temperature and total precipitation, respectively, over period g of scenario i. Note that the different periods, g, have different lengths and lag times but the event probabilities are all functions of a single set of weights, w_i, $i = 1, \ldots, n$. As written here in (2a–e), all scenarios are considered to contain period g, for all values of g. If this is not the case, then only those n scenarios that do contain period g, for all values of g, are used in practice. Eq. (2e) corresponds to the relative frequencies summing to unity. Redundant and nonintersecting (infeasible) equations must be eliminated so that the remaining m equations number less than or equal to n. If $m = n$, (2) can be solved via Gauss-Jordan elimination as a system of linear equations for the weights, w_i, since the equations would be independent and intersecting (in n-space). For $m < n$, there are multiple solutions, and identification of the "best" becomes an optimization problem; e.g., Croley (1996) suggests

$$\min \sum_{i=1}^{n} (w_i - 1)^2 \text{ subject to } \sum_{i=1}^{n} a_{k,i}w_i = e_k; \quad k = 1, \ldots, m \tag{3}$$

where "subject to" constraint equations = remaining m equations identified in (2) but rewritten in alternate form; $a_{k,i}$ = 0 or 1 corresponding to exclusion or inclusion, respectively, of each variable in the sets of (2); and e_k corresponds to event probabilities specified in the probabilistic meteorology outlook (e.g., $e_k = n\hat{P}[T_k \le \tau_{k,0.333}]$; $k = 1, \ldots, 14$).

MATCHING MOST-PROBABLE EVENTS

Consider matching most-probable event forecasts such as those available as NOAA's 6–10 day outlooks for average air temperature and total precipitation or EC's one-month and three-month outlooks for average air temperature. Most-probable event forecasts are a special case of a more general category of probability statements. Generally, $r + 1$ intervals for a variable's values are set by defining interval limits, $z_1 < z_2 < \cdots < z_r$. The general form of the probability statement, to which a most-probable event forecast can be cast, is that the jth event (interval) has a probability in excess of a specified value, written here in terms of the relative frequencies to be matched

$$\hat{P}[z_{j-1} < X \le z_j] > \phi_j \tag{4}$$

where X may be average air temperature or total precipitation; and ϕ_j = probability limit; $z_0 = -\infty$ and $z_{r+1} = +\infty$ are understood and, for these cases, (4) is defined to be

$$\hat{P}[z_0 < X \le z_1] = \hat{P}[X \le z_1] \tag{5a}$$

$$\hat{P}[z_r < X \le z_{r+1}] = \hat{P}[X > z_r] \tag{5b}$$

[In both the NOAA and EC forecasts of most-probable events, z_k is defined as the γ_k quantile (ξ_k) estimated from the 1961–90 period

$$\hat{P}[X \le \xi_k] = \gamma_k; \quad 1 \le k \le r \tag{6}$$

where $\gamma_1 < \gamma_2 < \cdots < \gamma_r$; and ϕ_k is defined in terms of quantile probabilities

$$\phi_k = \gamma_k - \gamma_{k-1}; \quad 1 \le k \le r + 1 \tag{7}$$

where $\gamma_0 = 0$ and $\gamma_{r+1} = 1$. For the NOAA 6–10 day most-probable event temperature forecast, $r = 4$, $\gamma_1 = 0.1$, $\gamma_2 = 0.3$, $\gamma_3 = 0.7$, and $\gamma_4 = 0.9$ ($\phi_1 = 0.1$, $\phi_2 = 0.2$, $\phi_3 = 0.4$, $\phi_4 = 0.2$, and $\phi_5 = 0.1$); for the NOAA 6–10 day most-probable event precipitation forecast and both the EC one- and three-month most-probable event temperature forecasts, $r = 2$, $\gamma_1 = 1/3$, and $\gamma_2 = 2/3$ ($\phi_1 = \phi_2 = \phi_3 = 1/3$). However, the more general definitions of z_k and ϕ_k are used herein to allow for other outlooks that may be more broadly defined than either of the present NOAA or EC most-probable event forecasts.]

Many most-probable event forecasts are implicitly accompanied by the assumption that only the most-probable interval has forecast probability exceeding its reference probability. Eq. (4) would then become

$$\hat{P}[z_{j-1} < X \le z_j] > \phi_j \tag{8a}$$

$$\hat{P}[z_{k-1} < X \le z_k] \le \phi_k; \quad k = 1, \ldots, r + 1; \quad k \ne j \tag{8b}$$

Alternatively, (8) can be written as

$$\hat{P}[\text{not}(z_{j-1} < X \le z_j)] < 1 - \phi_j \tag{9a}$$

$$\hat{P}[z_{k-1} < X \leq z_k] \leq \phi_k; \quad k = 1, \ldots, r + 1; \quad k \neq j \tag{9b}$$

If the assumption is not desired then the r equations in (9b) are omitted.

Weights are determined by matching relative frequencies, as in (1), to the most-probable interval forecasts of (9)

$$\frac{1}{n} \sum_{i \in E_j} w_i < 1 - \phi_j; \quad E_j \equiv \{i | \text{not}(z_{j-1} < x_i \leq z_j)\} \tag{10a}$$

$$\frac{1}{n} \sum_{i \in F_k} w_i \leq \phi_k; \quad F_k \equiv \{i | z_{k-1} < x_i \leq z_k\};$$

$$k = 1, \ldots, r + 1; \quad k \neq j \tag{10b}$$

Alternatively, we can write (10) as follows:

$$\sum_{i=1}^{n} a_{j,i} w_i < e_j \tag{11a}$$

$$\sum_{i=1}^{n} a_{k,i} w_i \leq e_k; \quad k = 1, \ldots, r + 1; \quad k \neq j \tag{11b}$$

where $a_{k,i}$ are defined similarly to (3) as 0 or 1 corresponding to exclusion or inclusion, respectively, of each variable in the sets of (10); and e_k corresponds to probability limits specified in the most-probable event forecast [$e_j = n(1 - \phi_j)$ and $e_k = n\phi_k$, $k \neq j$]. The $r + 1$ inequalities in (11) represent one most-probable event forecast; if we have multiple most-probable event forecasts (from different agencies, for different periods and lags, and for different variables), we represent them by the $p + q$ inequalities

$$\sum_{i=1}^{n} a_{k,i} w_i < e_k; \quad k = 1, \ldots, p \tag{12a}$$

$$\sum_{i=1}^{n} a_{k,i} w_i \leq e_k; \quad k = p + 1, \ldots, p + q \tag{12b}$$

where p = total number of "strictly less-than" constraints; and q = total number of "less-than-or-equal-to" constraints to be considered. Note that while (12) may refer to different variables over different periods with different lengths and lag times, the equations are written in terms of a single set of weights (w_i, $i = 1, \ldots, n$), as was done for (2).

MIXING PROBABILISTIC METEOROLOGY OUTLOOKS

By adding the constraints corresponding to most-probable event forecasts in (12) to those of the event probability forecasts in (3), we now have the optimization

$$\min \sum_{i=1}^{n} (w_i - 1)^2 \text{ subject to} \tag{13a}$$

$$\sum_{i=1}^{n} a_{k,i} w_i = e_k; \quad k = 1, \ldots, m \tag{13b}$$

$$\sum_{i=1}^{n} a_{k,i} w_i < e_k; \quad k = m + 1, \ldots, m + p \tag{13c}$$

$$\sum_{i=1}^{n} a_{k,i} w_i \leq e_k; \quad k = m + p + 1, \ldots, m + p + q \tag{13d}$$

which is equivalent to

$$\min \sum_{i=1}^{n} (w_i - 1)^2 \text{ subject to} \tag{14a}$$

$$\sum_{i=1}^{n} a_{k,i} w_i = e_k; \quad k = 1, \ldots, m \tag{14b}$$

$$\sum_{i=1}^{n} a_{k,i} w_i + w_{n+k-m} = e_k; \quad k = m + 1, \ldots, m + p + q \tag{14c}$$

$$w_i > 0; \quad i = n + 1, \ldots, n + p \tag{14d}$$

$$w_i \geq 0; \quad i = n + p + 1, \ldots, n + p + q \tag{14e}$$

where w_i, ($i = n + 1, \ldots, n + p + q$) = "slack" variables added to change consideration of inequality constraint to consideration of equality constraint in optimization. This, in turn, is equivalent to

$$\min \sum_{i=1}^{n} (w_i - 1)^2 \text{ subject to} \tag{15a}$$

$$\sum_{i=1}^{n+p+q} a_{k,i} w_i = e_k; \quad k = 1, \ldots, m + p + q \tag{15b}$$

$$w_i > 0; \quad i = n + 1, \ldots, n + p \tag{15c}$$

$$w_i \geq 0; \quad i = n + p + 1, \ldots, n + p + q \tag{15d}$$

where the additional coefficients are defined as follows:

$$a_{k,i} = 0; \quad k = 1, \ldots, m; \quad i = n + 1, \ldots, n + p + q \tag{16a}$$

$$a_{k,i} = 1; \quad k = m + 1, \ldots, m + p + q; \quad i = n + k - m \tag{16b}$$

$$a_{k,i} = 0; \quad k = m + 1, \ldots, m + p + q; \quad i > n, \quad i \neq n + k - m \tag{16c}$$

If we ignore the nonnegativity constraints ($w_i > 0$, $i = n + 1, \ldots, n + p$ and $w_i \geq 0$, $i = n + p + 1, \ldots, n + p + q$) for now, (15) becomes

$$\min \sum_{i=1}^{n} (w_i - 1)^2 \text{ subject to} \tag{17a}$$

$$\sum_{i=1}^{n+p+q} a_{k,i} w_i = e_k; \quad k = 1, \ldots, m + p + q \tag{17b}$$

which is similar to (3) and may be solved as before (Croley 1996) by defining the Lagrangian (Hillier and Lieberman 1969)

$$L = \sum_{i=1}^{n} (w_i - 1)^2 - \sum_{k=1}^{m+p+q} \lambda_k \left(\sum_{i=1}^{n+p+q} a_{k,i} w_i - e_k \right) \tag{18}$$

(where λ_k = unit penalty of violating kth constraint in optimization) and by setting the first derivatives with respect to each variable to zero

$$\frac{\partial L}{\partial w_i} = 2(w_i - 1) - \sum_{k=1}^{m+p+q} \lambda_k a_{k,i} = 0; \quad i = 1, \ldots, n \tag{19a}$$

$$\frac{\partial L}{\partial w_i} = -\sum_{k=1}^{m+p+q} \lambda_k a_{k,i} = 0; \quad i = n + 1, \ldots, n + p + q \tag{19b}$$

$$\frac{\partial L}{\partial \lambda_k} = -\sum_{i=1}^{n+p+q} a_{k,i} w_i + e_k = 0; \quad k = 1, \ldots, m + p + q \tag{19c}$$

We have a set of necessary but not sufficient conditions for the problem of (17). Eqs. (19a)–(19c) are linear and solvable via the Gauss-Jordan method of elimination. Sufficiency may be checked by inspection.

The solution of (17) may give positive, zero, or negative weights and slack variables, but only nonnegative or strictly positive weights (either $w_i \geq 0$ or $w_i > 0$, $i = 1, \ldots, n$) and slack variables ($w_i > 0$, $i = n + 1, \ldots, n + p$ and $w_i \geq 0$,

$i = n + p + 1, \ldots, n + p + q$) make physical sense, and we must further constrain the optimization. Two cases arise here

$$w_i > 0; \quad i = 1, \ldots, n \tag{20a}$$

$$w_i > 0; \quad i = n + 1, \ldots, n + p \tag{20b}$$

$$w_i \geq 0; \quad i = n + p + 1, \ldots, n + p + q \tag{20c}$$

and

$$w_i \geq 0; \quad i = 1, \ldots, n \tag{21a}$$

$$w_i > 0; \quad i = n + 1, \ldots, n + p \tag{21b}$$

$$w_i \geq 0; \quad i = n + p + 1, \ldots, n + p + q \tag{21c}$$

In both cases, we have a mixture of strictly positive ($w_i > 0$) and simply nonnegative ($w_i \geq 0$) weights and slack variables for the optimization. These additional constraints can result in infeasibility (there is no solution), and equations must be eliminated from (17b) to allow a feasible solution. To facilitate this, the engineer or hydrologist must prioritize the probabilistic meteorology outlook settings [and, hence, the equations in (17b)] so that the least important ones (lowest priority) can be eliminated first. The equation in (17b) corresponding to (2e) should always be given top priority.

A method of successive optimizations is depicted in the procedural algorithm of Fig. 2; it preserves as many of the probability settings as possible while yielding results identical to earlier methods when no slack variables are present (Croley 1996). In Fig. 2, if simple nonnegativity conditions would be violated in an optimization, even though positivity conditions may also be violated, the method adds a zero constraint ($w_i = 0$) for each negative variable ($w_i < 0$), as long as the resulting constraint set still represents a nonempty space, and re-solves the optimization. If the resulting constraint set would represent an empty solution space, then the method eliminates all earlier-added zero constraints and the lowest-priority probability setting instead and re-solves the optimization. If only positivity constraints would be violated, then the method simply eliminates all earlier-added zero constraints and the lowest-priority probability setting and re-solves the optimization. Two variations are depicted in Fig. 2. "Method 1" guarantees that only strictly positive weights will result, as in (20), and all possible future scenarios are used (no scenario is weighted by zero and effectively eliminated). "Method 2" disallows some of the possible future scenarios (by allowing zero weights), as in (21); this generally allows satisfaction of more event probability settings than does method 1.

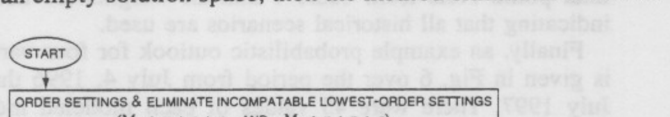

FIG. 2. Determining Physically Relevant Weights and Slack Variables

MIXED MULTIPLE OUTLOOKS EXAMPLE

In making a hydrological outlook on July 4, 1996, we have available probabilistic outlooks made on the following four separate dates: (1) the NOAA climate outlook for July 1996 (event probabilities for July air temperature and precipitation and for three-month air temperature and precipitation over 13 periods, successively lagged one month each, starting with July–August–September 1996) made June 13, 1996; (2) the NOAA 6–10 day outlook for July 9–13, 1996 (most-probable event for five-day air temperature and precipitation) made July 3, 1996; (3) the EC climate outlook for July 1996 (most-probable event for July 1996 air temperature) made July 1, 1996; and (4) the EC climate outlook for June–July–August 1996 (most-probable event for three-month air temperature) made June 1, 1996. Values for the Lake Superior basin are abstracted from these outlooks in Fig. 3. Note that the EC outlook for July 1996 in Fig. 3 is incompatible with the NOAA one-month outlook for the same period. Twenty-one settings are arbitrarily selected, shaded in Fig. 3, in the priority order indicated in Fig. 4, to make a hydrological outlook for Lake Superior beginning July 4, 1996. The priority order was set arbitrarily

Event Probabilities for July 1996 Air Temperature and Precipitation and for JAS 1996 through JAS 1997 Air Temperature and Precipitation, forecast 13 June 1996 by NOAA:

$\hat{P}[T_{July96} \le \tau_{July,0.333}] = 0.333$
$\hat{P}[T_{July96} > \tau_{July,0.667}] = 0.333$
$\hat{P}[Q_{July96} \le \theta_{July,0.333}] = 0.333$
$\hat{P}[Q_{July96} > \theta_{July,0.667}] = 0.333$
$\hat{P}[T_{JAS96} \le \tau_{JAS,0.333}] = 0.313$
$\hat{P}[T_{JAS96} > \tau_{JAS,0.667}] = 0.353$
$\hat{P}[Q_{JAS96} \le \theta_{JAS,0.333}] = 0.333$
$\hat{P}[Q_{JAS96} > \theta_{JAS,0.667}] = 0.333$
$\hat{P}[T_{ASO96} \le \tau_{ASO,0.333}] = 0.333$
$\hat{P}[T_{ASO96} > \tau_{ASO,0.667}] = 0.333$
$\hat{P}[Q_{ASO96} \le \theta_{ASO,0.333}] = 0.333$
$\hat{P}[Q_{ASO96} > \theta_{ASO,0.667}] = 0.333$
$\hat{P}[T_{SON96} \le \tau_{SON,0.333}] = 0.363$
$\hat{P}[T_{SON96} > \tau_{SON,0.667}] = 0.303$
$\hat{P}[Q_{SON96} \le \theta_{SON,0.333}] = 0.333$
$\hat{P}[Q_{SON96} > \theta_{SON,0.667}] = 0.333$
$\hat{P}[T_{OND96} \le \tau_{OND,0.333}] = 0.333$
$\hat{P}[T_{OND96} > \tau_{OND,0.667}] = 0.333$
$\hat{P}[Q_{OND96} \le \theta_{OND,0.333}] = 0.333$
$\hat{P}[Q_{OND96} > \theta_{OND,0.667}] = 0.333$
$\hat{P}[T_{NDJ96} \le \tau_{NDJ,0.333}] = 0.333$
$\hat{P}[T_{NDJ96} > \tau_{NDJ,0.667}] = 0.333$
$\hat{P}[Q_{NDJ96} \le \theta_{NDJ,0.333}] = 0.333$
$\hat{P}[Q_{NDJ96} > \theta_{NDJ,0.667}] = 0.333$
$\hat{P}[T_{DJF96} \le \tau_{DJF,0.333}] = 0.293$
$\hat{P}[T_{DJF96} > \tau_{DJF,0.667}] = 0.373$
$\hat{P}[Q_{DJF96} \le \theta_{DJF,0.333}] = 0.333$
$\hat{P}[Q_{DJF96} > \theta_{DJF,0.667}] = 0.333$
$\hat{P}[T_{JFM97} \le \tau_{JFM,0.333}] = 0.263$
$\hat{P}[T_{JFM97} > \tau_{JFM,0.667}] = 0.403$
$\hat{P}[Q_{JFM97} \le \theta_{JFM,0.333}] = 0.393$
$\hat{P}[Q_{JFM97} > \theta_{JFM,0.667}] = 0.273$
$\hat{P}[T_{FMA97} \le \tau_{FMA,0.333}] = 0.333$
$\hat{P}[T_{FMA97} > \tau_{FMA,0.667}] = 0.333$
$\hat{P}[Q_{FMA97} \le \theta_{FMA,0.333}] = 0.333$
$\hat{P}[Q_{FMA97} > \theta_{FMA,0.667}] = 0.333$
$\hat{P}[T_{MAM97} \le \tau_{MAM,0.333}] = 0.303$
$\hat{P}[T_{MAM97} > \tau_{MAM,0.667}] = 0.363$
$\hat{P}[Q_{MAM97} \le \theta_{MAM,0.333}] = 0.333$
$\hat{P}[Q_{MAM97} > \theta_{MAM,0.667}] = 0.333$
$\hat{P}[T_{AMJ97} \le \tau_{AMJ,0.333}] = 0.333$
$\hat{P}[T_{AMJ97} > \tau_{AMJ,0.667}] = 0.333$
$\hat{P}[Q_{AMJ97} \le \theta_{AMJ,0.333}] = 0.333$
$\hat{P}[Q_{AMJ97} > \theta_{AMJ,0.667}] = 0.333$
$\hat{P}[T_{MJJ97} \le \tau_{MJJ,0.333}] = 0.333$
$\hat{P}[T_{MJJ97} > \tau_{MJJ,0.667}] = 0.333$
$\hat{P}[Q_{MJJ97} \le \theta_{MJJ,0.333}] = 0.333$
$\hat{P}[Q_{MJJ97} > \theta_{MJJ,0.667}] = 0.333$
$\hat{P}[T_{JJA97} \le \tau_{JJA,0.333}] = 0.333$
$\hat{P}[T_{JJA97} > \tau_{JJA,0.667}] = 0.333$
$\hat{P}[Q_{JJA97} \le \theta_{JJA,0.333}] = 0.333$
$\hat{P}[Q_{JJA97} > \theta_{JJA,0.667}] = 0.333$
$\hat{P}[T_{JAS97} \le \tau_{JAS,0.333}] = 0.333$
$\hat{P}[T_{JAS97} > \tau_{JAS,0.667}] = 0.333$
$\hat{P}[Q_{JAS97} \le \theta_{JAS,0.333}] = 0.333$
$\hat{P}[Q_{JAS97} > \theta_{JAS,0.667}] = 0.333$

Most-Probable Event for 9-13 July 1996 Air Temperature and Precipitation, forecast 3 July 1996 by NOAA:

$\hat{P}[T_{9-13July96} \le \tau_{9-13July,0.100}] \le 0.100$
$\hat{P}[\tau_{9-13July,0.100} < T_{9-13July96} \le \tau_{9-13July,0.300}] > 0.200$
$\hat{P}[\tau_{9-13July,0.300} < T_{9-13July96} \le \tau_{9-13July,0.700}] \le 0.400$
$\hat{P}[\tau_{9-13July,0.700} < T_{9-13July96} \le \tau_{9-13July,0.900}] \le 0.200$
$\hat{P}[T_{9-13July96} > \tau_{9-13July,0.900}] \le 0.100$

$\hat{P}[Q_{9-13July96} \le \theta_{9-13July,0.333}] \le 0.333$
$\hat{P}[\theta_{9-13July,0.333} < Q_{9-13July96} \le \theta_{9-13July,0.667}] > 0.334$
$\hat{P}[Q_{9-13July96} > \theta_{9-13July,0.667}] \le 0.333$

Most-Probable Event for July 1996 Air Temperature, forecast 1 July 1996 by EC:

$\hat{P}[T_{July96} \le \tau_{July,0.333}] > 0.333$
$\hat{P}[\tau_{July,0.333} < T_{July96} \le \tau_{July,0.667}] \le 0.334$
$\hat{P}[T_{July96} > \tau_{July,0.667}] \le 0.333$

Most-Probable Event for JJA 1996 Air Temperature, forecast 1 June 1996 by EC:

$\hat{P}[T_{JJA96} \le \tau_{JJA,0.333}] \le 0.333$
$\hat{P}[\tau_{JJA,0.333} < T_{JJA96} \le \tau_{JJA,0.667}] > 0.334$
$\hat{P}[T_{JJA96} > \tau_{JJA,0.667}] \le 0.333$

FIG. 3. Lake Superior Probabilistic Meteorology Outlooks Available July 4, 1996

for this example to reflect user confidence in the settings, with the earliest, shortest outlooks given highest priority. Other priority orders are possible, of course, and are discussed subsequently. Note in Fig. 4 that the settings corresponding to "greater-than" inequalities in Fig. 3 were rewritten to be "less-than" inequalities as indicated in (9). These 21 outlook settings are used with inspection of the 45 12-month time series, beginning July 4 from the available historical record of 1948–93, to construct 22 equations represented by (17*b*) in Fig. 5. The first row in Fig. 5 corresponds to (2*e*), wherein all weights sum to the number of scenarios (45 in this example). Rows 2–10 in Fig. 5 correspond to the nine inequalities at the

$$\hat{P}\left[Q_{9-13July96} \le \theta_{9-13July,0.333}\right] \le 0.333 \qquad \text{(NOAA 6-10d)}$$
$$\hat{P}\left[\text{not}\left(\theta_{9-13July,0.333} < Q_{9-13July96} \le \theta_{9-13July,0.667}\right)\right] < 1-0.334 \qquad \text{(NOAA 6-10d)}$$
$$\hat{P}\left[Q_{9-13July96} > \theta_{9-13July,0.667}\right] \le 0.333 \qquad \text{(NOAA 6-10d)}$$
$$\hat{P}\left[\text{not}\left(T_{July96} \le \tau_{July,0.333}\right)\right] < 1-0.333 \qquad \text{(EC 1m)}$$
$$\hat{P}\left[\tau_{July,0.333} < T_{July96} \le \tau_{July,0.667}\right] \le 0.334 \qquad \text{(EC 1m)}$$
$$\hat{P}\left[T_{July96} > \tau_{July,0.667}\right] \le 0.333 \qquad \text{(EC 1m)}$$
$$\hat{P}\left[T_{JJA96} \le \tau_{JJA,0.333}\right] \le 0.333 \qquad \text{(EC 3m)}$$
$$\hat{P}\left[\text{not}\left(\tau_{JJA,0.333} < T_{JJA96} \le \tau_{JJA,0.667}\right)\right] < 1-0.334 \qquad \text{(EC 3m)}$$
$$\hat{P}\left[T_{JJA96} > \tau_{JJA,0.667}\right] \le 0.333 \qquad \text{(EC 3m)}$$
$$\hat{P}\left[T_{JAS96} \le \tau_{JAS,0.333}\right] = 0.313 \qquad \text{(NOAA 3m)}$$
$$\hat{P}\left[T_{JAS96} > \tau_{JAS,0.667}\right] = 0.353 \qquad \text{(NOAA 3m)}$$
$$\hat{P}\left[T_{SON96} \le \tau_{SON,0.333}\right] = 0.363 \qquad \text{(NOAA 3m)}$$
$$\hat{P}\left[T_{SON96} > \tau_{SON,0.667}\right] = 0.303 \qquad \text{(NOAA 3m)}$$
$$\hat{P}\left[T_{DJF96} \le \tau_{DJF,0.333}\right] = 0.293 \qquad \text{(NOAA 3m)}$$
$$\hat{P}\left[T_{DJF96} > \tau_{DJF,0.667}\right] = 0.373 \qquad \text{(NOAA 3m)}$$
$$\hat{P}\left[T_{JFM97} \le \tau_{JFM,0.333}\right] = 0.263 \qquad \text{(NOAA 3m)}$$
$$\hat{P}\left[T_{JFM97} > \tau_{JFM,0.667}\right] = 0.403 \qquad \text{(NOAA 3m)}$$
$$\hat{P}\left[Q_{JFM97} \le \theta_{JFM,0.333}\right] = 0.393 \qquad \text{(NOAA 3m)}$$
$$\hat{P}\left[Q_{JFM97} > \theta_{JFM,0.667}\right] = 0.273 \qquad \text{(NOAA 3m)}$$
$$\hat{P}\left[T_{MAM97} \le \tau_{MAM,0.333}\right] = 0.303 \qquad \text{(NOAA 3m)}$$
$$\hat{P}\left[T_{MAM97} > \tau_{MAM,0.667}\right] = 0.363 \qquad \text{(NOAA 3m)}$$

FIG. 4. Selected Outlook Probability Settings in Priority Order

k[a]	Weight Coefficient, $a_{k,i}$, $i = 1, \ldots, 54$[b]: Scenario Weights	Slack Variables	e_k[c]
(1)	(2)	(3)	(4)
1	111111111111111111111111111111111111111111111	000000000	1.000 × 45
2	110101110001100010000100001100001101000001100	100000000	0.333 × 45
3	110111111011100111100101101100101111001111100	010000000	0.666 × 45
4	000010001010000101100001100000100010001110000	001000000	0.333 × 45
5	110011110101010110100010011111011111101111100	000100000	0.667 × 45
6	100011100000010000000000001001101111010100100	000010000	0.334 × 45
7	010000010101000110100010001100000001000111000	000001000	0.333 × 45
8	001100000010000001011001100001100010010000001	000000100	0.333 × 45
9	011101010011010101111011100111100011110110011	000000010	0.666 × 45
10	010001010001010100100010000110000001100110010	000000001	0.333 × 45
11	001100101000001001010001101001000010010000001	000000000	0.313 × 45
12	110011010001110100100110010000000001000111010	000000000	0.353 × 45
13	000100000000100000110000010101001100001100010	000000000	0.363 × 45
14	100001101010111100001011010001000000110000000	000000000	0.303 × 45
15	000000001010011010100001010010100100000000000	000000000	0.293 × 45
16	100111101010001010010000010000110100011001111	000000000	0.373 × 45
17	010000000010001010100101010001100100000000000	000000000	0.263 × 45
18	000111000100100100000000010001000101011100111	000000000	0.403 × 45
19	000000011111101000000110000000100001101100011	000000000	0.393 × 45
20	111011100000000000110001100111010000000011000	000000000	0.273 × 45
21	010001010001000010100111011000100000000010000	000000000	0.303 × 45
22	000000100100010000010000100010001000111101101	000000000	0.363 × 45

[a]Priority (1 ≡ highest).

[b]Coefficients in (17b) defined for each selected probability setting, k, of the climate outlooks, and for each scenario, i, in the historical record ($i = 1, \ldots, 45$) or for each slack variable ($i = 46, \ldots, 54$).

[c]Probability of setting multiplied by the number of scenarios in the historical record ($n = 45$).

FIG. 5. Boundary Condition Eqs. (17*b*) for July 4, 1996 Lake Superior Outlook

TABLE 1. Climate Outlook Weights Using all Outlook Settings[a]

Year (1)	Weight (2)	Year (3)	Weight (4)
1948	0.056668	1971	0.893248
1949	0.690534	1972	1.790025
1950	0.702190	1973	1.310190
1951	1.084638	1974	1.023160
1952	0.251568	1975	0.682992
1953	0.389880	1976	1.568554
1954	0.238341	1977	0.857767
1955	0.893465	1978	1.093807
1956	1.136410	1979	1.396049
1957	0.978764	1980	1.372525
1958	1.080701	1981	1.027267
1959	1.407542	1982	0.690876
1960	0.841958	1983	0.866815
1961	1.295717	1984	1.342744
1962	0.947023	1985	1.757465
1963	0.629091	1986	1.492841
1964	1.385337	1987	0.430635
1965	0.817966	1988	1.032797
1966	1.446744	1989	1.095609
1967	1.300806	1990	0.825888
1968	0.782530	1991	1.177490
1969	1.102380	1992	0.845085
1970	0.965916		

[a]Solution of (17) with Fig. 5 coefficients and method 2 in Fig. 2; all outlook settings in Fig. 4 are used.

top of Fig. 4, wherein slack variables ($w_{46}, \ldots, w_{54}$) were added to convert the inequalities into equations. Rows 11–22 in Fig. 5 correspond to the 12 equations at the bottom of Fig. 4. Table 1 presents the solution of these equations, found by minimizing the deviation of weights from unity, as in (17), by utilizing all 21 climate outlook settings (method 2 in Fig. 2). All computations were made with probabilities (both reference quantiles and forecasts) significant to three digits after the decimal point. Note from Table 1 that all weights are nonzero, indicating that all historical scenarios are used.

Finally, an example probabilistic outlook for four variables is given in Fig. 6 over the period from July 4, 1996 through July 1997. There were 45 values of each modeled monthly variable (basin moisture, runoff, lake temperature, and lake evaporation), over each of the 12 full months and one partial month (July 1996), corresponding to the 45 scenarios used in the simulation. Each of these 52 samples of 45 values was used with the weights in Table 1 in (1) to compute nonparametric distributions for the probabilistic outlooks.

METHODOLOGY CONSIDERATIONS

An artifact of the methodology pictured in Fig. 2 is that the elimination of a couple of constraints from a previous solution (i.e., expanding the constraint space of the optimization) could result in the methodology eliminating even more constraints in its new search for a solution (optimum solution to the expanded constraint space). That is, if we had an optimum solution to a set of constraints that also satisfied (20) or (21), it theoretically should be possible to eliminate a couple of constraints, redo the optimization, and then find the new optimum solution no longer satisfies (20) or (21). In this case, the continuing methodology of Fig. 2 would eliminate the lowest-priority constraint and re-solve the optimization until an optimum solution is identified that also satisfies (20) or (21). Since the solution to a set of constraints also satisfies any subset of those constraints, we might be surprised that the methodology of Fig. 2 could possibly continue searching for a solution by eliminating even more constraints, after we had broadened the constraint space by removing some constraints. While unobserved so far, this should be possible. It would

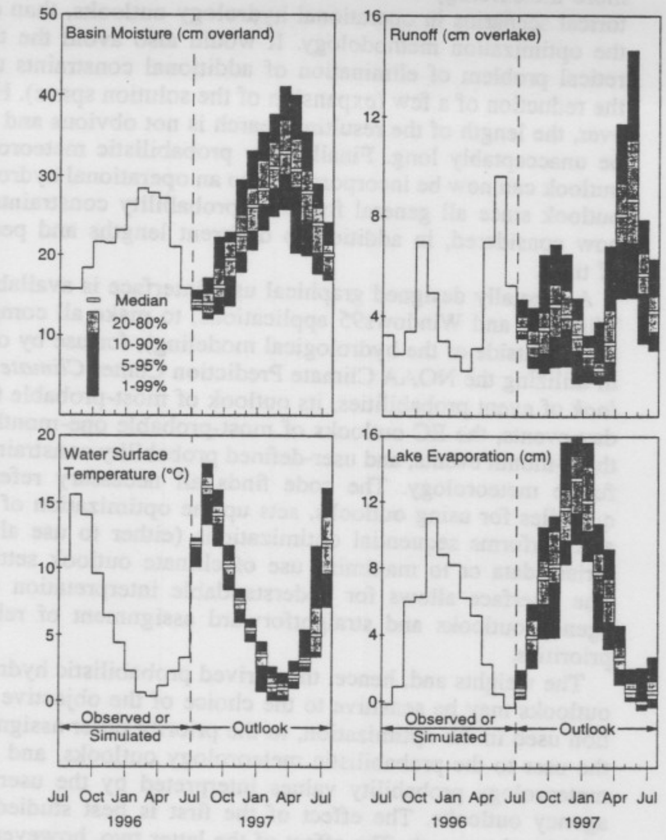

FIG. 6. Example Lake Superior Hydrological Outlooks, July 4, 1996

$$\hat{P}[T_{July96} \le \tau_{July,0.333}] = 0.333$$
$$\hat{P}[T_{July96} > \tau_{July,0.667}] = 0.333$$
$$\hat{P}[T_{JAS96} \le \tau_{JAS,0.333}] = 0.313$$
$$\hat{P}[T_{JAS96} > \tau_{JAS,0.667}] = 0.353$$
$$\hat{P}[T_{ASO96} \le \tau_{ASO,0.333}] = 0.333$$
$$\hat{P}[T_{ASO96} > \tau_{ASO,0.667}] = 0.333$$
$$\hat{P}[T_{9-13July96} \le \tau_{9-13July,0.100}] \le 0.100$$
$$\hat{P}[\tau_{9-13July,0.100} < T_{9-13July96} \le \tau_{9-13July,0.300}] > 0.200$$
$$\hat{P}[\tau_{9-13July,0.300} < T_{9-13July96} \le \tau_{9-13July,0.700}] \le 0.400$$
$$\hat{P}[\tau_{9-13July,0.700} < T_{9-13July96} \le \tau_{9-13July,0.900}] \le 0.200$$
$$\hat{P}[T_{9-13July96} > \tau_{9-13July,0.900}] \le 0.100$$

FIG. 7. Difficult-to-Satisfy Temperature Outlook Probability Settings in Priority Order

result because the methodology only finds an optimum solution, which might not satisfy (20) or (21), and disregards other feasible solutions to the constraint equations. There are always other feasible solutions (combinations of weights that satisfy all the constraints) that are not optimum. We are interested in those solutions too; the optimization in Fig. 2 is only a device to find a solution that might also satisfy (20) or (21). Unfortunately, systematic searches of the constraint space in (17) for feasible solutions (not necessarily optimum) that also satisfy (20) or (21) involve evaluation of numerous roots, which is computationally impractical. If we could formulate an acceptance criterion for usable solutions (not necessarily optimum), then evaluation of all solutions is unnecessary and we might build a partial search algorithm that is practical. Again, however, there is not an obvious way to guarantee that the length of the resulting search is acceptably short.

Another problem in either the sequential optimization approach used here or in a partial search algorithm, just described, is the selection of a priority order. Priorities may be assigned according to user confidence in the meteorology outlooks, user goals or purposes for which the hydrological outlooks will be used (e.g., February air temperatures may be much more important for snow melt events than June–July–August precipitation), or something other. Other priority orders may give satisfaction of more equations (Croley 1996). For example, if the first 32 equations identified in Fig. 3 are used in the priority of their appearance there, then the algorithm of Fig. 2 [used to satisfy (21), (method 2)] gives weights that satisfy the first 30 of those equations, when used with the 45 scenarios of the available historical Lake Superior basin meteorology record. Alternatively, if the priorities of these 32 equations are reversed, then the first four equations in Fig. 3 are unused. Elimination of other than lowest-priority equations would lead to alternate solutions too.

Inspection of the constraints is always a good idea, to avoid mixing settings that may be difficult to satisfy simultaneously. Elimination of a difficult-to-satisfy setting, which is of marginal interest to a user, may allow the satisfaction of more probability setting constraints in method 1 or the use of more scenarios in method 2. For example, consider using the first six temperature event probabilities from the June 13, 1996 forecast by NOAA and the most-probable temperature event from NOAA's July 3, 1996 forecast from Fig. 3 in their order of appearance there. These are summarized in Fig. 7. If using method 1 (to guarantee all scenarios are used) in the operational hydrology analysis, then the last four equations in Fig. 7 would be unused (eliminated in the algorithm of Fig. 2) with the Lake Superior basin data. Alternatively, by eliminating only the second equation in Fig. 7, all remaining probability setting constraints would be used. (Equivalently, we could have placed the second equation in Fig. 7 last in the priority structure where it then would have been eliminated.) This is because the second equation and the last five inequalities in Fig. 7 may be difficult (but not impossible) to satisfy simultaneously, depending on the historical meteorology of the available scenarios. That is, it may be difficult to get a uniform distribution for July temperature with an increased probability of low temperature for July 9–13. If method 2 is used (to maximize the number of probability setting constraints used in Fig. 7), the same thing happens in this example.

Of course, it is also important to eliminate truly incompatible settings, as indicated in the first block of the algorithm of Fig. 2 and as was done in the example of Figs. 4–6 and Table 1. As noted previously, the EC outlook for July 1996 in Fig. 3 is incompatible with the NOAA one-month outlook for the same period. In particular, the following two equations are incompatible:

$$\hat{P}[T_{July\,96} \le \tau_{July,0.333}] = 0.333; \quad \text{(NOAA event probability)} \qquad (22a)$$

$$\hat{P}[T_{July\,96} \le \tau_{July,0.333}] > 0.333; \quad \text{(EC most-probable event)} \qquad (22b)$$

The NOAA one-month outlook was eliminated prior to the computation of weights. In the consideration of the four types of outlooks portrayed in Fig. 3, there may be, in general, truly incompatible settings only between the NOAA one-month event probabilities and the EC one-month most-probable event, or only between the NOAA three-month event probabilities and the EC three-month most-probable event; these are easily checked by inspection.

EXTENSIONS

The formulation of an optimization problem allows for a general approach in determining operational hydrology weights in the face of multiple outlooks where many solutions are possible but difficult to systematically evaluate. In the absence of a partial search algorithm for finding and evaluating other (than optimum) solutions (feasible weight combinations), one could modify the optimization objective function. For example, replace (17*a*) for method 1 with

$$\min\left[\sum_{i=1}^{n}(w_i-1)^2+\sum_{i=1}^{n+p}f(w_i)+\sum_{i=n+p+1}^{n+p+q}g(w_i)\right] \tag{23}$$

or for method 2 with

$$\min\left[\sum_{i=1}^{n}(w_i-1)^2+\sum_{i=1}^{n}g(w_i)+\sum_{i=n+1}^{n+p}f(w_i)+\sum_{i=n+p+1}^{n+p+q}g(w_i)\right] \tag{24}$$

where

$$f(w)=M;\quad w\le 0 \tag{25a}$$

$$f(w)=0;\quad w>0 \tag{25b}$$

$$g(w)=M;\quad w<0 \tag{25c}$$

$$g(w)=0;\quad w\ge 0 \tag{25d}$$

and M = very large number. Minimization would force positive or nonnegative solutions if they exist. However, these formulations are not amenable to the techniques employed here in terms of defining a Lagrangian function that is continuous, and allowing linear equations that are solvable via the Gauss-Jordan method of elimination.

Note that any meteorological probability constraint can be incorporated into a hydrological outlook because it must be of one of the following general forms:

$$\hat{P}[z_1<X\le z_2]=a \tag{26a}$$

$$\hat{P}[z_1<X\le z_2]>a \tag{26b}$$

$$\hat{P}[z_1<X\le z_2]<a \tag{26c}$$

$$\hat{P}[z_1<X\le z_2]\ge a \tag{26d}$$

$$\hat{P}[z_1<X\le z_2]\le a \tag{26e}$$

and all of these forms, or their converses, are considered in the development of the algorithm in Fig. 2. Of course, if the user adds additional probability constraints, he or she must also check for incompatibilities within the entire constraint set.

SUMMARY AND CONCLUSIONS

Today's probabilistic meteorology outlooks of event probabilities or most-probable events can be mixed in operational hydrology outlooks so that hydrological forecasts match meteorological outlooks. This mixing is accomplished by expressing all meteorology outlooks as equality or inequality constraints in an optimization, converting the problem to an equivalent set of equations, and solving them similar to an earlier method for considering only equations (event probabilities). The reformulation of the earlier method requires modification of the earlier method algorithms, but yields the same results when applied to only event probabilities. The mixed multiple outlooks example for July 4, 1996 on the Lake Superior basin illustrates the method and the importance of prioritizing the meteorology outlooks.

Results depend on priority order and on identification of meteorology outlooks that are difficult-to-satisfy or truly incompatible. The methodology may be extended by redefining the optimization objective function to allow direct consideration of nonnegativity or positivity constraints; however, an alternate solution would be required. Alternatively, formulation of an acceptance criterion, for usable solutions that are not necessarily optimum, would enable a partial search algorithm to identify more solutions. This could allow for satisfaction of more meteorology outlooks or allow for the use of more historical scenarios in operational hydrology outlooks, than does the optimization methodology. It would also avoid the theoretical problem of elimination of additional constraints upon the reduction of a few (expansion of the solution space). However, the length of the resulting search is not obvious and may be unacceptably long. Finally, any probabilistic meteorology outlook can now be incorporated into an operational hydrology outlook since all general forms of probability constraints are now considered, in addition to different lengths and periods of time.

A specially designed graphical user interface is available as Windows and Windows95 applications, to make all computations (outside of the hydrological modeling), for use by others in utilizing the NOAA Climate Prediction Center *Climate Outlook* of event probabilities, its outlook of most-probable 6–10 day events, the EC outlooks of most-probable one-month and three-month events, and user-defined probability constraints on future meteorology. The code finds all necessary reference quantiles for using outlooks, sets up the optimization of (17), and performs sequential optimizations (either to use all historical data or to maximize use of climate outlook settings). The interface allows for understandable interpretation of all agency outlooks and straightforward assignment of relevant priorities.

The weights and, hence, the derived probabilistic hydrology outlooks may be sensitive to the choice of the objective function used in the optimization, to the priority order assigned by the user to the probabilistic meteorology outlooks, and to the meteorology probability values interpreted by the user from agency outlooks. The effect of the first is best studied with additional research. The effect of the latter two, however, may be assessed by users in their own applications, by simply repeating all calculations with alternate priority assignments or probability values. This is greatly facilitated by the graphical user interface. A recomputation of weights and their application to make probabilistic hydrology outlooks does not require re-creating the hydrological scenarios to which to apply the weights.

ACKNOWLEDGMENT

This paper is GLERL contribution no. 1023.

APPENDIX I. REFERENCES

Croley II, T. E. (1996). "Using NOAA's new climate outlooks in operational hydrology." *J. Hydrologic Engrg.*, ASCE, 1(3), 93–102.

Croley II, T. E. (1997). "Water resource predictions from meteorological probability forecasts." *Sustainability of water resources under increasing uncertainty (Proc., Rabat Symp.)*, D. Rosbjerg et al., ed., IAHS Publication No. 240, IAHS Press, Wallingford, U.K., 301–310.

Day, G. N. (1985). "Extended streamflow forecasting using NWSRFS." *J. Water Resour. Plng. and Mgmt.*, ASCE, 111, 157–170.

Hillier, F. S., and Lieberman, G. J. (1969). *Introduction to operations research—appendix 2: classical optimization techniques*. Holden-Day, San Francisco, Calif.

Smith, J. A., Day, G. N., and Kane, M. D. (1992). "Nonparametric framework for long-range streamflow forecasting." *J. Water Resour. Plng. and Mgmt.*, ASCE, 118(1), 82–92.

APPENDIX II. NOTATION

The following symbols are used in this paper:

A_g = set of indices of scenarios containing average air temperature for period g in lower third of its 1961–90 range;

$a_{k,i}$ = coefficient in kth constraint equation on ith weight (for ith scenario) or slack variable;

B_g = set of indices of scenarios containing average air temperature for period g in upper third of its 1961–90 range;

C_g = set of indices of scenarios containing total precipitation for period g in lower third of its 1961–90 range;

D_g = set of indices of scenarios containing total precipitation for period g in upper third of its 1961–90 range;
E_j = set of indices of scenarios containing values of X for period i (x_i) not within interval, $(z_{j-1}, z_j]$;
e_k = selected weights sum limit in kth equation, corresponding to event probabilities specified in probabilistic meteorology outlook;
F_k = set of indices of scenarios containing values of X for period i (x_i) within interval, $(z_{k-1}, z_k]$;
$f(\ )$ = penalty associated with nonpositive argument (large number used in objective function minimization to avoid nonpositive values);
$g(\ )$ = penalty associated with negative argument (large number used in objective function minimization to avoid negative values);
L = objective function (Lagrangian) for unconstrained optimization reformulated from objective function for constrained optimization by incorporating constraints;
m = number of probability constraint equations selected from probabilistic meteorological outlooks for use in making operational hydrology outlook;
n = number of scenarios (number of historical record segments) available for use in generating operational hydrology outlook;
$\hat{P}[\ ]$ = relative frequency in set, of event in brackets, used as probability estimate;
p = number of "strictly less-than" probability constraint inequalities selected from probabilistic meteorological outlooks for use in making operational hydrology outlook;
Q_g = total precipitation over period g;
q = number of "less-than-or-equal-to" probability constraint inequalities selected from probabilistic meteorological outlooks for use in making operational hydrology outlook;
$q_{g,i}$ = total precipitation over period g of scenario i;
r = number of interval limits defining $r + 1$ intervals of real line for variable's values;
T_g = average air temperature over period g;
$t_{g,i}$ = average air temperature in period g of scenario i;
w_i = weight to apply to ith value of X (x_i) in set of possible future scenarios, or added "slack variable" to convert inequality to equation;
X = meteorological or hydrological variable;
x_i = value for variable X in ith scenario in set of n possible future scenarios;
z_k = kth interval limit ($k = 1, \ldots, r$) defining $r + 1$ intervals of real line for variable's values;
$\theta_{g,\gamma}$ = reference total precipitation γ-probability quantile for period g;
λ_k = Lagrange multiplier, representing unit penalty associated with violation of kth constraint equation in optimization;
ξ_k = reference γ-probability quantile for variable X;
$\tau_{g,\gamma}$ = reference average air temperature γ-probability quantile for period g;
ϕ_j = probability limit for most-probable interval outlook for interval j; and
Ω = set of indices of scenarios.